ADHERENT JET ALONG A CURVED SURFACE

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The problem of a flat jet traveling along a curved surface has been solved numerically and the results are discussed here. The characteristics of a developing boundary layer as well as the frictional stresses at a convex and at a concave surface are shown.

Let a solid curved surface be placed in an infinite space filled with a viscous fluid. Off this surface there is an infinitesimally narrow discharging orifice oriented parallel to the surface. A jet of fluid flows from this orifice tangent to the surface, the physical properties of this fluid being the same as those of the ambient fluid. The jet travels a long distance enveloping the surface. This is to say that the pressure drop from the outer filled space to the vicinity of the surface balances the forces of inertia.

Thus, the flow is governed principally by the pressure drop across and along the boundary layer. The significance of the longitudinal pressure drop is evident inasmuch as the jet velocity decreases farther away from the surface while the pressure at the source increases correspondingly and approaches the ambient pressure. Therefore, the classical boundary-layer equations, which are used for analyzing the jet flow along a flat wall, cannot be applied here, because they do not account for the transverse pressure gradient.

We will introduce an orthogonal curvilinear system of coordinates with the origin at the jet source. The x-coordinates run along the surface in the direction of the jet, the y-axis is normal to the surfaces. The equations of motion for an incompressible fluid are in this case [1]

$$\begin{gathered}\frac{u}{1\pm y/R}\cdot\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\pm\frac{uv}{R(1\pm y/R)}=-\frac{1}{\rho(1\pm y/R)}\cdot\frac{\partial p}{\partial x}\\ +\nu\left[\frac{\partial^2 u}{\partial y^2}\pm\frac{1}{R(1\pm y/R)}\cdot\frac{\partial u}{\partial y}-\frac{u}{R^2(1\pm y/R)^2}\right];\\ \frac{1}{\rho}\cdot\frac{\partial p}{\partial y}=\pm\frac{u^2}{R(1\pm y/R)};\qquad \frac{\partial u}{\partial x}+\frac{\partial}{\partial y}[(1\pm y/R)v]=0\end{gathered}\tag{1}$$

and the boundary conditions are

$$u=0,\quad v=0\quad \text{at}\quad y=0;\quad u=0\quad \text{at}\quad y\to\infty. \tag{2}$$

The plus sign in system (1) corresponds to a convex surface, the minus sign here corresponds to a concave surface. In calculating the jet discharge along a curved surface, autonomous solutions have been considered which are feasible if the radius of curvature is a specified monomial power function of the longitudinal coordinate x [2, 3].

Autonomous solutions do not exist in the general case, however, because there are two parameters in the problem. A method leading to a quasiautonomous solution is based on expanding the flow function into a power series in terms of the perturbation parameter, but this is valid only near the source [4]. We, therefore, propose here a numerical solution.

On the basis of the incompressibility condition, we express the flow function $\Psi(x, y)$ as

$$\Psi(x,\ y)=\sqrt[4]{E_0\nu x}\,F(\eta,\ \xi);\ \eta=y\sqrt[4]{\frac{E_0}{\nu^3x^3}};\quad \xi=\frac{x\nu}{\sqrt[3]{E_0R^4}},\tag{3}$$

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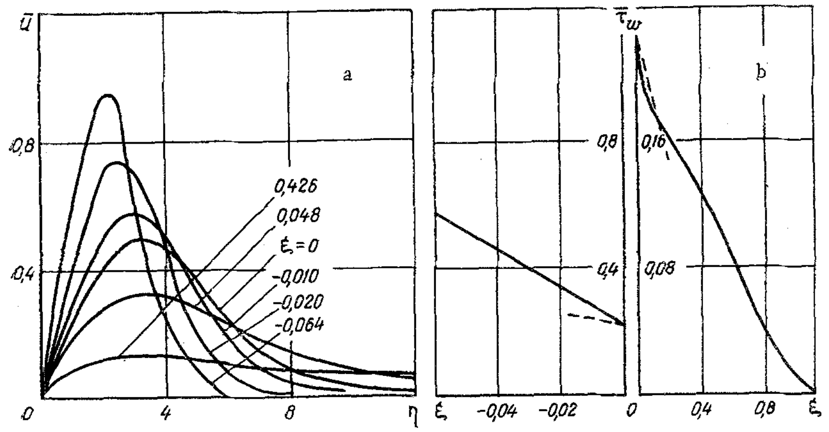

Fig. 1. Dimensionless velocity profile $\bar{u}(a)$ and frictional stress $\bar{\tau}_w$(b) as functions of the ξ-coordinate.

where the numerical constant E_0 is not the same as the product of the volume flow per second and the momentum per unit mass in [5]. Then Eq. (1) for function F will become

$$\frac{\partial^3 F}{\partial\eta^3}+\frac{1}{2}\left(\frac{\partial F}{\partial\eta}\right)^2+\frac{1}{4}F\frac{\partial^2 F}{\partial\eta^2}=\xi\left(\frac{\partial F}{\partial\eta}\cdot\frac{\partial^2 F}{\partial\xi\partial\eta}-\frac{\partial^2 F}{\partial\eta^2}\cdot\frac{\partial F}{\partial\xi}\right)$$
$$\mp\frac{\xi^{3/4}}{1\pm\eta\xi^{3/4}}\cdot\frac{\partial F}{\partial\eta}\left(\frac{1}{4}F+\xi\frac{\partial F}{\partial\xi}-\frac{3}{4}\eta\frac{\partial F}{\partial\eta}\right)$$
$$\mp 2\xi^{3/4}\int\limits_{\eta}^{\infty}\frac{\partial F}{\partial\eta}\left(\frac{-\frac{1}{2}\cdot\frac{\partial F}{\partial\eta}-\frac{3}{4}\eta\frac{\partial^2 F}{\partial\eta^2}+\xi\frac{\partial^2 F}{\partial\xi\partial\eta}}{1\pm\eta\xi^{3/4}}\right)d\eta\mp\eta\xi^{3/4}\frac{\partial^3 F}{\partial\eta^3}\mp\xi^{3/4}\left(\frac{\partial^2 F}{\partial\eta^2}\mp\frac{\xi^{3/4}}{1\pm\eta\xi^{3/4}}\cdot\frac{\partial F}{\partial\eta}\right), \tag{4}$$

with the upper sign referring to a convex surface and the lower sign referring to a concave surface.

The boundary conditions (2) are now rewritten as

$$F=0,\quad \frac{\partial F}{\partial\eta}=0 \quad \text{at} \quad \eta=0;\quad \frac{\partial F}{\partial\eta}=0 \quad \text{at} \quad \eta\to\infty. \tag{5}$$

The resulting Eq. (4) with the boundary conditions (5) is universal when referred to a constant radius of curvature and it needs to be solved only once. The introduced function $F(\eta, \xi)$ satisfies the Akatnov equation [5] at $\xi = 0$ and, thus, the initial profile at $\xi = 0$ is represented as a solution to the Akatnov equation for a jet traveling along a flat wall.

By introducing the function $\varphi = \partial F/\partial\eta$, we reduce Eq. (4) to a second-order differential equation which has been solved numerically. The numerical solution to Eq. (4) with the boundary conditions (5) was obtained by the implicit difference scheme. The derivatives with respect to η and ξ were replaced by central differences and the resulting system of nonlinear algebraic equations was solved on each ξ-step by simple iterations, while the linearized system of difference equations was solved in each iteration by the elimination method [6].

The results of these computations to obtain a dimensionless velocity profile $\bar{u} = u/\sqrt{E_0/\nu x}$ and a dimensionless profile of frictional stress $\bar{\tau}_w = \tau_w/\mu\sqrt[4]{E_0^3/\nu^5 x^5}$ are shown in Fig. 1a, b respectively.

The profiles of velocity and frictional stress depend largely on the curvature of the surface. As ξ increases, the velocity profile at a concave surface becomes more peaked with a maximum which also increases and shifts toward lower values of the autonomous variable η. The frictional stress follows an analogous trend. At a convex surface the jet slows down and at some section the boundary layer separates from the surface. Numerical calculations indicate that the critical section is determined by the parameter value $\xi_{cr} = 1.15$. At higher values of ξ the profile becomes blurred.

An essential feature here is that a restructuring of the boundary layer occurs within the initial segment along the curved surface (Fig. 1). The results obtained in [4] are also shown in Fig. 1b.

An explanation for the discrepancies is that in [4] the flow function was expanded into series in terms of a small parameter, valid only near the source. Only two terms of the series were retained there, and this resulted in large errors in calculations of the flow far away from the source.

NOTATION

x, y	are the longitudinal and transverse coordinate respectively;
u, v	are the projections of the velocity vector on the normals of tangents to the x- and y-coordinate curves respectively;
$\Psi(x, y)$	is the flow function;
η, ξ	are the dimensionless coordinates;
$F(\eta, \xi)$	is the dimensionless flow function;
τ_w	is the tangential skin-friction stress;
ρ	is the density;
ν	is the kinematic viscosity;
μ	is the dynamic viscosity;
R	is the radius of curvature.

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